ANALYSIS OF THE TRANSFER CHARACTERISTICS OF A PUSH-PULL TYPE MAGNETIC AMPLIFIER

By

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LIST OF SYMBOLS AND SUBSCRIPTS

Symbols	Description
e	Instantaneous voltage (volts)
E, V	Average voltage (volts)
i	Instantaneous current (amperes)
I	Average current (amperes)
f	Frequency (cycles per second)
R, r	Resistance (ohms)
Z	Impedance (ohms)
t	Time (seconds)
T	Time constant (seconds)
N	Number of turns per leg
ϕ	Magnetic flux (webers)
B	Flux density (webers per sq. m.)
	Firing angle (electrical degree)

Subscripts	Description
C	Control winding
B	Bias winding
F	Feedback winding
L	Load winding
i	Input value, initial value
o	Output value
f	Forward value
s	Saturated value
m	Maximum value

CHAPTER I

INTRODUCTION

RECENT DEVELOPMENT OF MAGNETIC-AMPLIFIER CIRCUITS.- The magnetic amplifier is a rather new device. It has been used in master gun stabilizers, automatic pilot and ground approach systems, blind landing aid and to regulate fuel flow in some types of guided missles. During the past five years, the subject of magnetic amplifiers has undergone considerable development, not only in a continuously widening range of industrial and military applications, but also in the addition of important advances in fundamental theory.

Push-pull type magnetic amplifiers are especially useful in the field of electrical measurements and in high performance servomechanisms, including the control of two-phase induction motors. There is obviously a very close linkage between the various types of magnetic amplifiers, because similar saturable core arrangements and similar circuit techniques are used. The wide variety of typical magnetic amplifier circuits, especially the push-pull type, makes difficult their extensive illustration.

INTRODUCTION TO THE PROBLEM.- The magnetic amplifier, through rather simple in external form, is highly complex in the internal mechanics of its operation. The nonlinear nature of

the magnetic amplifier introduces many difficulties not ordinarily encountered in electric machinery, thus rendering work with magnetic amplifiers the more fascinating. The saturable core of a magnetic amplifier has a severe nonlinear characteristic, and the hysteresis loop of the core is imperfectly known under dynamic conditions. The threshold effect of rectifiers introduces another nonlinearity. In actual operation. the rectifier, because of its nonlinear characteristics, does not have appreciable conduction at low values of alterating voltage.

The push-pull type magnetic amplifier considered in this paper (see Figure 1) consists of four saturable cores with an external differential feedback winding and with d-c signal and rectified bias. This type magnetic amplifier supplies duodirectional output, and performs precise measurements in high performance feedback loop systems. Other variations are also in use and can be analyzed similarly.

In the analysis, simplifications and assumptions are required at various stages. For example, in order to simplify the analysis, one may assume operation free from even-harmonic currents or with suppressed even-harmonic currents. One may assume all rectifier stacks with constant reverse and forward resistances or with infinite reverse resistance and zero forward resistance. One may assume that the hysteresis loops of the magnetic cores are ideal square curves or with finite

slope during the exciting period. Compared with the experimental data, some analytic results obtained by the assumption of an idealized square hysteresis loop were fairly good. Ideal square hysteresis loop material is nonexistent, but we can assume the ideal. All these assumptions are necessary in order to make the mathematics manageable. The philosophy behind this is that an answer slightly incorrect is better than no answer at all, and one can hope that, if all these assumptions are fairly good, the final answer will be fairly close to the truth.

PURPOSE AND SCOPE OF STUDY. - The problems involved in the use of magnetic amplifiers in push-pull are numerous. Among these are a great many which, to the author's knowledge, have not been investigated completely. A four-core push-pull type magnetic amplifier with external feedback has extremely useful characteristics. Applications of this circuit are appearing in increasing numbers in the literature, but until recently an analytical study was not available because of complexity of the feedback loops and the interactions between cores in their operation.

The problem of circuit analysis is that of examining a given circuit configuration. As a result of the analysis, one may predict the operating sequences and modes in different signal conditions, the requirement of the control signal for optimum operation, and whether the circuit would perform

more satisfactorily if some change in the circuit elements or voltages were made. Working with the given circuit, certain assumptions will be made in order to simplify the process of analysis without significantly alternating the results. The analysis is limited to steady-state operation. For a transient study the labor involved would be large, and the core information needed should be very comprehensive since many dynamic modes occur throughout the flux reset.

Since the circuit to be discussed is very complicated, the analysis will start from the study of the individual building blocks of the circuit and then combine these building blocks and study their interactions by describing with simple mathematics the relationships between the sinusoidal supply voltage, firing angles, core fluxes, signal current and output current.

FUNDAMENTAL THEOREMS USED IN THE ANALYSIS. - There are many rules used in magnetic amplifier theory. Among these rules two very important ones may be recognized as fundamental theorems. The first theorem is called the "average flux theorem." The second one is called the "symmetry theorem."

The Average Flux Theorem. - This theorem is based upon the elementary physical fact that in steady-state operation of magnetic amplifiers the flux in the core must have a finite average value. For periodic excitation, the core flux value at time α/w must be identical to that at $(2\pi+\alpha)/w$. In other

words, the flux change in a core during any complete period is zero. Since core flux equals the integral of voltage with respect to time, the mathematical representation of the average flux theorem is

$$\int_{\theta_0}^{\theta_0+2\pi} e\,d(wt) = 0 \ .$$

<u>The Symmetry Theorem.</u> - This theorem is widely used by various authors in magnetic amplifier theory. The theorem can be concisely stated as follows :

For a magnetic amplifier consisting of two identical saturable cores, core 1 and core 2, in steady-state operation the gate winding voltage of core 1 in the nth half-cycle is identical to the gate winding voltage of core 2 in the (n+1)th half-cycle. In other words, in the (n+1)th half-cycle core 2 repeats the function of core 1 in the nth half-cycle, or vice versa.

The mathematical representation of the symmetry theorem is

$$e_1(wt) = e_2(wt+\pi)$$

or

$$e_2(wt) = e_1(wt+\pi)$$

This theorem is valid for series amplifiers, parallel amplifiers and the entire group of the so-called "self-saturating amplifiers".

If one considers these two theorems together, from their mathematical representations the following result can easily be obtained;

$$\int_{\theta_0}^{\theta_0+2\pi}(e_1 - e_2)d(wt) = 0.$$

This equation has important physical significance. The graphical display of the voltage difference to be integrated gives a clear picture of the mechanism by which the amplifier is controlled by the signal voltage. The firing angles may be found from this equation as a function of signal voltage. The equation is valid for steady-state only. For the transient state, the integral is not equal to zero; it is equal to the flux gain or flux loss of the core during a complete period.

PRACTICAL DATA ON CIRCUIT ELEMENTS. - The following data are taken from a four-core push-pull type magnetic amplifier.

Core material: $\frac{1}{2}$x0.002" thick, supermalloy tape.

Core size: diameters, $1\frac{3}{8}$" and $1\frac{3}{4}$".

Load winding: 4000 turns #34 wire, 130 ohms.

Bias winding: 2000 turns #34 wire, 130 ohms.

Feedback winding: 2000 turns #34 wire, 100 ohms.

Control winding: 2000 turns #34 wire, 130 ohms.

Transformer: Core material: $\frac{1}{2}$x0.002" orthonol tape.

Core size: diameters, $1\frac{1}{4}$" and 2.0".

Primary winding: 4000 turns #33 wire, 230 ohms.

Secondary winding: 2000 turns #33 wire, 100 ohms.

Rectifiers: Diffused-junction germanium. (GE type 1N93)

CHAPTER II

BASIC BUILDING BLOCKS OF PUSH-PULL TYPE MAGNETIC AMPLIFIER

SINGLE-ENDED MAGNETIC AMPLIFIER.- The circuit to be analyzed in this paper is shown in Figure 1.[(1)*] This circuit is composed of two identical single-ended magnetic amplifiers. It is convenient first to discuss the modes of operation and transfer characteristics of the single-ended magnetic amplifier.

A single-ended magnetic amplifier is shown in Figure 2. It consists of two identical saturable reactors with d-c control windings connected in series and bias winding excited by a rectified voltage.

The operation of the single-ended magnetic amplifier is characterized by two distinct modes of operation distributed in time. The first mode is called the control mode, and the second mode is called the output mode. The effect of the positive feedback is to increase the quiescent current of the amplifier. The defect can be eliminated if two single-ended magnetic amplifiers connected in push-pull with differential feedback are used as in the circuit of Figure 1. Figure 3 shows the transfer characteristic of the single-ended magnetic amplifier.

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*Superscript numerals refer to bibliography.

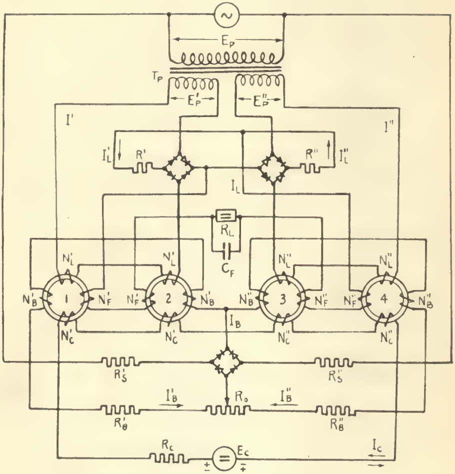


Fig. 1. Push-pull type magnetic amplifier with differential-feedback. (W. A. Geyger.)

OPERATION ANALYSIS OF THE BASIC MAGNETIC AMPLIFIER. - For simplicity, the circuit to analyzed is the configuration shown in Figure 5a instead of that in Figure 2.

During steady state operation, the only changes between successive half-cycles are the polarity of e_p and the direction of the current in the gate winding. In the (n+1)th half-cycle, core 1 repeats the operation of core 2 in the nth half-cycle, or vice versa. For this reason only one-half cycle of operation will be analyzed; the other will be implied.

In the analysis, an ideal square magnetization curve is assumed; windings are assumed to have zero resistance; circuit elements are assumed to be linear. Also assume a low impedance control winding, a pure resistance load,and idealized rectifiers.

During mode 1 operation (gating period), the exciting current is negligibly small; thus the voltage drop across resistors may be neglected if compared with the voltage across the core winding. In other words, all the supplied voltage is absorbed by the core to build the core flux.

During mode 2 operation (output period), the voltage across the winding is negligibly small if compared with the voltage drop across the resistors.

Using these two conditions and the average flux theorem and symmetry theorem, the circuit equations of signal

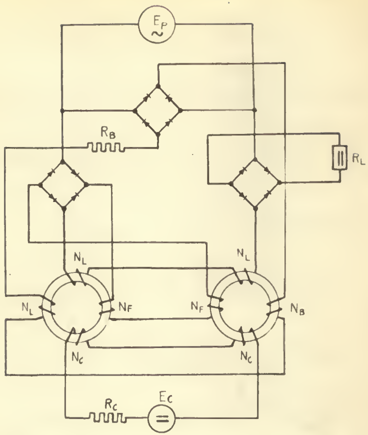
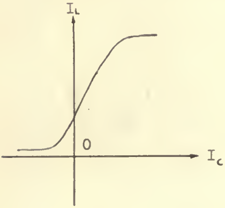
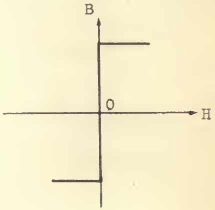


Fig. 2. Single-ended magnetic amplifier.



(3)



(4)

Fig. 3. Transfer characteristic of single-ended magnetic amplifier.

Fig. 4. Ideal hysteresis loop.

current, load current and firing angle can be obtained.

Modes of Operation.- Define the following modes of operation:

Mode 1. Neither core saturated.

Mode 2. One core saturated.

The polarity shown in Figure 5a is defined as the nth half-cycle, positive signal.

At the beginning of the nth half-cycle, core 2 starts from a higher flux level than core 1; i.e., this flux had set into it on the previous half-cycle. Flux in core 2 will be built up to saturation during the nth half-cycle operation. At the beginning of the (n-1)th half-cycle, as the gate winding current reverses direction and the sum of the mmfs of core 2 becomes zero, the output period is over and the core returns to its unsaturated condition. In any core, the operation of the nth half-cycle is identical to the operation of the (n-2)th half-cycle.

During mode 1 operation, since the exciting current is negligibly small, the voltages across R , R and r are negligible if compared with the voltage drops across the core windings.

During mode 1 operation, the loop equations of the circuit are

$$e_p = e_1 + e_2 - \frac{N_F}{N_L}e_1 + \frac{N_F}{N_L}e_2$$

$$E_c = \frac{N_c}{N_L}e_2 - \frac{N_c}{N_L}e_1$$

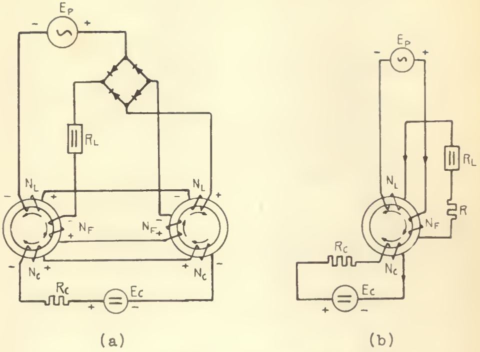


Fig. 5 (a). Polarities of the magnetic amplifier during the nth half-cycle operation.

(b). Equivalent circuit during mode 2.

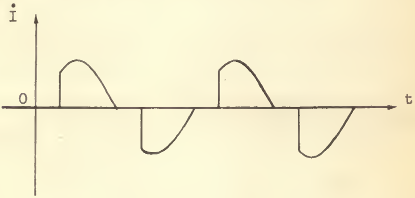


Fig. 6. Theoretical waveform of the output current in load winding.

Where e_1 and e_2 are voltages across load winding N_L of core 1 and core 2 respectively. Solve for e_1 and e_2 in terms of e_p and E_c:

$$e_1 = \frac{e_p}{2} - \frac{N_L + N_F}{2N_c} E_c \qquad (1)$$

$$e_2 = \frac{e_p}{2} + \frac{N_L - N_F}{2N_c} E_c \qquad (2)$$

Voltage applied to the load winding N_L is induced into the control winding; this induced voltage is in the order of magnitude of the supply voltage times the turns-ratio. If $N_L = N_F$, the net mmf due to N_L and N_F, during the exciting period of core 1, is zero; thus only the control winding supplies magnetizing force to core 1. Since the magnetizing forces applied to core 2 aid each other and core 2 starts from a higher flux level, it will saturate and core 1 will remain unsaturated during the nth half-cycle of operation.

After core 2 becomes saturated, voltages across its windings drop to a negligibly small value; currents flow through the load and control windings and operation changes from mode 1 to mode 2.

During mode 2 operation, the circuit loop equations become

$$E_c = i_c R_c - \frac{N_c}{N_L} e_1' \qquad (3)$$

$$e_p = i_L (R_L + R') + e' - \frac{N_F}{N_L} e_1' \qquad (4)$$

For $N_F = N_L$, $e_p = i_L(R_L + R')$.

Once core 2 is saturated, current flows through the core windings. Core 1 remains unsaturated throughout mode 2 operation. Its function is similar to that of a current transformer; i.e. the sum of the ampere-turns of core 1 will be zero. Thus the following relation holds throughout mode 2 operation.

$$i_L N_L = i_L N_F + i_c N_c \qquad (5)$$

The control current i_c consists of a d-c component and a even-harmonic component. If the voltage drop due to the even-harmonics is negligibly small, then $E_c \doteq i_c R_c$ and consequently $e_i' = 0$. Thus after core 2 saturates, the flux level in core 1 will remain constant. The value of load current and control current are

$$i = \frac{e_p}{R_L + R'}, \quad \text{and} \quad i_c = \frac{E_c}{R_c}.$$

If we solve for load current and control current directly from equations (3), (4) and (5) the following relation are obtained:

$$i_L = \frac{N_c^2 e_p + (N_L - N_F) N_c E_c}{R\,(N_L - N_F)^2 + N_c^2 (R_L + R')} \qquad (6)$$

$$i_c = \frac{(N_L - N_F)}{N_c} i_L \qquad (7)$$

The ampere-gain of the magnetic amplifier is:

$$\frac{I_L}{I_c} = \frac{N_c}{N_L - N_F} \qquad (8)$$

This equation implies that if unity feedback is used, the ampere-gain of the amplifier will be infinity. Hence, unity feedback leads to instability.

The equivalent circuit of the magnetic amplifier in mode 2 operation is shown in Figure 5b.

The "average flux theorem" shows that the flux change of a compete cycle in the steady-state is zero. Since the flux change is the voltage-time integral, the average voltage of the load winding during a complete cycle is zero. This description gives a clear picture of the flux reset phenomena. The firing angle, as a function of control voltage, may be found by this flux reset relation.

The "average flux theorem" gives the following equation:

$$\int_0^{\alpha_1} e_1\,dt + \int_{\alpha_1}^{\pi} e_1'\,dt + \int_{\pi}^{\pi+\alpha_1} e_2\,dt = 0$$

Where e_1, e_2 are given in equations (1) and (2) respectively.

If the even-harmonic component is free flowing in the control circuit, then, $e_1' = 0$, Thus the equation becomes

$$\int_0^{\alpha_1} e_1\,dt + \int_{\pi}^{\pi+\alpha_1} e_2\,dt = 0 .$$

The quench-fire angle is assumed to be zero in the equation. This assumption is very close to the truth; as soon as the gate winding current reverses direction, the saturated core will drop to the unsaturated condition. The

steady-state firing angle may be found by performing the foregoing integration.

TRANSCONDUCTANCE OF THE BASIC MAGNETIC AMPLIFIER. - Since the low impedance control circuit is chosen, the load current contains only odd-harmonic currents. The theoretical waveform of the load current is shown in Figure 6. The equation of the load current is

$$i_L = I_m \sin nwt \left[U(t-\alpha_1) - U(t-\pi) + U(t-\pi-\alpha_1) - U(t-2\pi) + \cdots\right]$$

The average load current is

$$I_L = \frac{E_p}{(R_L+R')}\int_{\alpha_1}^{\pi} \sin nwt\, d(wt) = \frac{E_p}{(R_L+R')}(1-\cos\alpha_1) \quad (9)$$

The transfer function of the magnetic amplifier in steady-state operation may be found by the following process: Let the Laplace transform of the first period be $F_1(s)$. Then the Laplace transform of the second period is $F_1(s)e^{-\frac{\pi}{\omega}s}$.

. . . .

The Laplace transform of the nth period is $F_1(s)e^{-\frac{(n-1)\pi}{\omega}s}$. Adding these we get the transform of the function:

$$F(s) = F_1(s)\left[1+e^{-\frac{\pi}{\omega}s}+e^{-\frac{2\pi}{\omega}s}+ \cdots +e^{-\frac{(n-1)\pi}{\omega}s}+\cdots\right]$$

$$= F_1(s)/(1-e^{-\frac{\pi}{\omega}s}).$$

where

$$F(s) = \mathcal{L}\left[\frac{E_p}{R'+R_L}\sin nwt\left\{U(t-\alpha_1)-U(t-\pi)\right\}\right]$$

$$= \frac{E_p}{R'+R_L}\int_{\frac{\alpha_1}{\omega}}^{\frac{\pi}{\omega}} \sin nwt\; e^{-st}dt = \frac{E_p}{R'+R_L}\,\frac{2W}{S^2+W^2}\left(e^{-\frac{\alpha_1}{\omega}s}-e^{-\frac{\pi}{\omega}s}\right)$$

The Laplace transform of the output current is then

$$F(S) = \frac{2E_p W}{(R_L + R')(S^2 + W^2)} \left(\frac{e^{-\frac{\alpha_1}{\omega}s} - e^{-\frac{\pi}{\omega}s}}{1 - e^{-\frac{\pi}{\omega}s}} \right) \qquad (10)$$

The circuit transconductance is defined by

$g = I_L / E_c$.

When the input is a constant d-c voltage, the output current is the wave shown in Figure 6.

BLOCK DIAGRAM OF THE BASIC MAGNETIC AMPLIFIER. - The feedback phenomenon in an ordinary magnetic amplifier is very complicated. In a practical magnetic core the hysteresis loop deviates from the ideal square hysteresis loop. There is a small exciting current flowing in the load and feedback winding during the control period. The transfer functions of magnetic amplifiers with and without feedback are different. The transfer function, which transforms the feedback current into the control circuit, is applied during the saturation and exciting interval.

During the nth half-cycle operation, the loop equation of the load winding is

$$E \sin nwt = (N_F - N_L) \frac{d\phi_1}{dt} - (N_F + N_L) \frac{d\phi_2}{dt}$$

In the case of no even-harmonic effect in the control winding, then

$e_1 = e_2$,

$$\frac{d\phi_1}{dt} = \frac{d\phi_2}{dt}$$

The maximum flux swing in the core (from preset flux level to saturation) is

$$\Delta\phi = \frac{1}{2N_L}\int_0^{\alpha_1} \frac{E_m}{2w} \sin wt\, d(wt) = \frac{E_m}{4\pi fN}\left(\frac{1-\cos\alpha_1}{2}\right) \qquad (11)$$

The average output voltage may be found from Eq. 9:

$$E_o = I_L(R' + R_L) = \frac{E_m}{\pi}(1 - \cos\alpha_1) \qquad (12)$$

Rewriting Eq. 11,

$$\Delta\phi = \frac{1}{N_L}\,\frac{E_o}{8f} \qquad (13)$$

Following Storm's method,[2] during the transient state consider the incremental variable only:

$$N_c\frac{d\phi}{dt} + R_c\bar{i}_c = \bar{E}_c$$

or

$$N_c\frac{d}{dt}\left(\frac{\bar{E}_o}{8fN_L}\right) + \frac{\bar{E}_o}{K_E} = \bar{E}_c$$

where $K_E = \frac{dE_o}{dE_c}$, the steady-state voltage gain.

Let $\frac{d}{dt} = s$; then

$$\frac{N_c}{N_L}\,\frac{\bar{E}_o}{8f}\, s + \bar{E}_o\frac{1}{K_E} = \bar{E}_c$$

or

$$\frac{E_o}{E_i} = \frac{K_E}{1 + sT} \qquad (14)$$

The block diagram of the basic magnetic amplifier is shown in Figure 7. The transfer function of the magnetic amplifier relates the change in output voltage E_o to the change in control voltage E_c . In this case E_o is very simply related to I_L by means of the load resistance R_L .

LOAD MIXING CIRCUIT

There are three types of load mixing circuits frequently used. They are the series mixing circuit, the parallel mixing circuit and the twin-winding mixing circuit. The last one is the most efficient way to get push-pull action but its usage is limited to a special load condition, such as is found in the first stage of a cascade push-pull magnetic amplifier. The parallel mixing circuit is chosen in the circuit under consideration.

Higher efficiency may be obtained if the resistors in the mixing circuit are properly chosen. For maximum power transfer, all the resistors in the circuit should be matched. Suppose core 1 saturates and the magnetic amplifier supplies power to the load. Let the saturated impedance of unit 1 be Z_s ; during this output period the impedance of the first unit is approximately fixed whereas the impedance of the second unit still remains infinity. Then the equivalent circuit of the magnetic amplifier may be represented by Figure 8a. Referring to the simplified circuit of Figure 8b, it is seen that in order to get maximum power transfer, the impedances

measured at the junctions aa' and bb' must be matched.

For matched impedance:

$$Z_{aa'} = Z_s = R' + \frac{R'R_L}{R' + R_L}$$

$$Z_{bb'} = R_L = \frac{R'(Z_s + R')}{Z_s + 2R'}$$

Eliminate Z_s from these two equations and solve for R_L:

$$R_L = \frac{2R'^2 + 3R'R_L}{3R' + 4R_L} \quad \text{or} \quad R_L = \frac{R'}{\sqrt{2}} \qquad (15)$$

Eliminate R' from these two equations and solve for Z_s:

$$Z_s = \sqrt{2}R_L + \frac{\sqrt{2}R_L^2}{(1+\sqrt{2})R_L} = 2R_L \qquad (16)$$

From equations (15) and (16), eliminate R_L:

$$Z_s = 2R' \qquad (17)$$

Equations (15), (16) and (17) give the matched loop impedances. If the circuit is matched, the total circuit impedance looking into the power supply terminals is:

$$Z_s + R' + \frac{R'R_L}{R' + R_L} = 2R_L + \sqrt{2}R_L + \frac{\sqrt{2}R_L^2}{(1+\sqrt{2})R_L} = 4R_L \qquad (18)$$

In the actual circuit, the value of the ballast resistor is usually chosen to be one to two times that of the load resistor.

The given circuit, in common with most push-pull circuits, is very inefficient. For this reason push-pull magnetic amplifiers are usually used in low power applications.

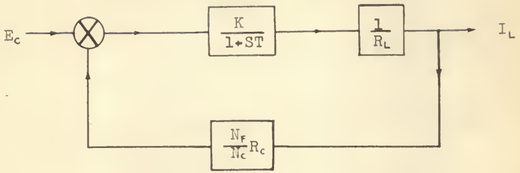
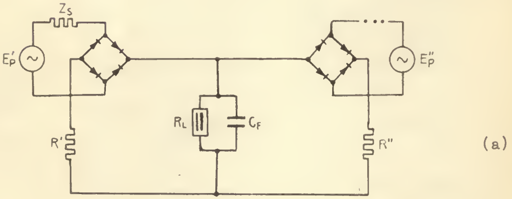


Fig. 7. Block diagram of single-ended magnetic amplifier.



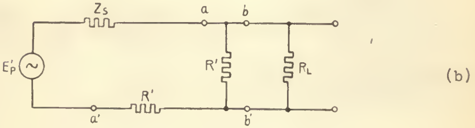


Fig. 8a. Equivalent circuit of the given push-pull type magnetic amplifier during mode 2 operation.

8b. Simplified version of the circuit of Fig. 8a.

CHAPTER III

PUSH-PULL CIRCUIT ANALYSIS

INTRODUCTION TO THE PUSH-PULL CIRCUIT PROBLEMS.- Referring to the asymmetrical transfer characteristic of Figure 3, it is evident that a positive control current in the basic magnetic amplifier causes an increase of load current, while a negative control current causes a decrease of load current. The load current changes only magnitude, not direction, when the control signal reverses polarity.

To obtain an external feedback type magnetic amplifier having duodirectional transfer characteristics, the best way is to connect two single-ended magnetic amplifiers in push-pull.

The response characteristic of the push-pull type magnetic amplifier may be represented by the curve b in Fig. 13. The response characteristic shows that the load current changes direction when the control signal reverses polarity.

With no signal, there is no output to the load. The variations of supply voltage and frequency do not affect the output current in the quiescent condition, if the selected circuit elements of these two single-ended units are completely matched. The quiescent currents flowing through the ballast resistors may vary considerably due to variations of the supply voltage and frequency, but the load current is free from zero-drift. The circuit elements must be carefully

matched over the working range to avoid excessive asymmetry zero-drift error. If these two units are not matched very well, a small change in one system will cause excessive change in the zero output point of operation. This change may be due to temperature, voltage or frequency variations. When two single-ended magnetic amplifiers are connected in push-pull, a power transformer is needed for electrical isolation between them. If the design of the power transformer is improper, it can materially affect the output characteristics.

ILLUSTRATION OF THE CIRCUIT CONSTRUCTION.- The circuit under consideration consists of two identical single-ended magnetic amplifier units, utilizing d-c signal, rectified bias excitation and differential type feedback. The characteristics of the single-ended magnetic amplifier units and the function of the load mixing circuit have been discussed in Chapter 2. The behavior of other elements used in the circuit will now be discussed.

(a)Bias Winding: The bias winding used in the saturable reactor produces additional d-c mmf. This bias mmf is independent of the control signal. Furthermore, bias excitation produces a shifting to the left of the response characteristic and thus offers a way to get a push-pull connection. In the normal working range, the magnetizing force of the bias winding must

be higher than that of the control winding, otherwise the output current will decrease with an increase of control signal. The value of the resistor R_8 is properly chosen so that it makes the saturable core fire at 90° in the quiescent condition. The slide-wire resistor R_6 is provided for the purpose of adjusting the current ratio to get a forcing balance between the two single-ended units. Zero-drift error can easily be accomplished by adjusting the slide-wire resistor. In practice, it is very difficult to manufacture two units completely matched; the slide-wire resistor is used to compensate for a slight mismatch due to manufacturing variations.

(b)<u>Control Winding</u>: The control windings of the four saturable cores are connected in such a way that the induced voltage at the fundamental frequency and odd-harmonics cancels out. Thus only even-harmonic voltage may be induced into the control winding. According to the reponse characteristic, a positive control current i increases the current I' but decreases the current I''. Even if the control signal increases without limit, there is a limit of maximum output. At the maximum output condition, one unit fires at zero angle and the other unit fires at 180°. After the limit condition, the second unit saturates in the same direction as the first one, and the output current begins to decrease with a further increase of control signal. At infinite control signal,

the output will drop to zero. The maximum output occurs when the magnetizing force of the control winding equals that of the bias winding.

(c) Feedback Winding: The positive feedback cancels out the intrinsic feedback of the saturable reactor. In order to increase the power amplification of the magnetic amplifier, external feedback is usually used. The given circuit uses the differential feedback method. The feedback current is the difference of two currents I' and I"; i.e., it is the actual load current I_L. The feedback effect is directly proportional to the load current. During the quiescent condition, no feedback current flows in the feedback winding; evidently, differential feedback does not have the defect of boosting the quiescent current. In the given circuit the feedback windings are wound in such a way that the magnetizing forces always aid the magnetizing force of the control winding. Due to the function of the feedback winding a very large gain is obtained and the signal input power is substantially reduced. The power output equation derived by Johannessen[3] is in the following form:

$$E_o\,(n+1) = \frac{N_L + N_F}{N_c}\;E_c(n) + E_o(n)$$

The terms $E_o(n)$ and $E_o(n+1)$ are the respective output of nth and (n+1)th half-cycles and $E_c(n)$ is the control voltage of the nth half-cycle. For any type magnetic amplifier,

the output on the $(n+1)$th half-cycle is a function of the output on the nth half-cycle. The equation shows that the amplifier is unstable if the coefficient of the feedback factor is unity. Unity feedback in this case is referred to as 100-percent positive feedback. However, if the magnetic amplifier supplies an inductive load, instability (a snap action) may occur for a feedback factor considerably smaller than unity.

(d)<u>Derivative Feedback</u>: In order to reduce the response time of the given circuit, a large capacity condenser is connected across the load resistor R_L. The parallel connected condenser acts as a time-variable series impedance on the feedback winding circuit. During the transient period, the load winding current tends to change rapidly and a large part of the transient load current can easily pass through the condenser; thus a relatively large feedback current flows through the feedback winding. By the derivative feedback method, the time constant of the given circuit can easily be reduced to about one fifth of its original value.(1)

<u>METHOD OF APPROACH OF THE ANALYSIS</u>.- The classical writings of H. F. Storm(2) on elementary magnetic amplifier theory make use of several simplifying assumptions. Storm chooses a square hysteresis loop and succeeds in describing with very simple mathematics the relationships between the sinusoidal

supply voltage, the firing angle, core fluxes, signal current and load current.

The operation of many of the circuits is critically dependent upon the details of the dynamic hysteresis loop, and as Finze, Lord and Critchow[4] and many others have pointed out, this loop depends on the speed of the magnetizing force variations, the width and shape of the past history of magnetization of the core, and other factors. Johannessen[3] introduced a successful method of analysis in which he used the half-cycle average values as the variable, and used the technique of difference equations to relate the output of any one half-cycle to the output of the preceding half-cycle.

The operational behavior of a single-ended magnetic amplifier has been discussed in Chapter 2. The effects of interaction between these two units will be considered in this chapter. Since the control windings are connected in series, any voltage induced in one of the core windings is inserted in series with the control voltage. When the first unit saturates, a voltage of $I'R_L$ is inserted into the load winding circuit of the second unit. This inserted voltage boosts the second unit applied voltage. In the analysis the following assumptions are used:

(1) Ideal square hysteresis loop; i.e., the differential permeability is very high during the unsaturated period and drops to a very low value after saturation. The ideal hysteresis

loop is shown in Figure 4.

(2) Idealized rectifier stacks, with infinite reverse resistance and zero or constant forward resistance.

(3) The two single-ended units are completely matched; corresponding elements are identical. The power transformer is large enough so that load effects may be neglected.

(4) A pure resistance load is chosen. A low impedance control circuit is assumed.

When a d-c signal is applied to the given circuit, it alters the voltage distribution between these two single-ended units and changes their firing angles. The firing angle of the saturable reactor is a function of time represented in terms of electrical degrees. The analysis will proceed by writing system equations and then solving these equations to get the input and output relations. During steady-state operation, successive unsaturated and saturated states occur in sequence, each saturation occuring in a definite angular interval. The problem remaining is to join these intervals to form an operation sequence and to find the relationships between input, output, winding voltage and firing angle. In the analysis of steady state operation, the control voltage and the control winding resistance appear as parameters in the solutions.

(4) Modes of operation. The circuit of Figure 1 carries the polarity-reversible d-c control signal in such a way that

push-pull action is obtained. When the control signal becomes zero, these two single-ended units are excited with identical magnetizing force, and thus they will fire at the same angle and supply the same output currents i' and i". The net load current ($\dot{I}_L = i' - i''$) becomes zero.

An equivalent circuit of the circuit of Figure 1 is shown in Figure 9. The polarities of the voltages of the load winding and the control winding are defined for the nth half-cycle and positive control signal. The arrow-head in the iron core shows the direction of flux flow.

The operation modes are defined as follows:

Mode 1. Neither core saturated.

Mode 2. One core saturated.

Mode 3. Two cores saturated.

During the nth half-cycle of operation, core 2 will saturate first, say at angle α_1, and core 3 will saturate at angle α_2. From α_1 to α_2 the amplifier supplies power to the load resistor R_L. Only mode 2 operation is the output period. Mode 1 and mode 3 do not deliver power to the load. The coercive forces of the magnetic cores with positive signal during the nth half-cycle of operation are

$$F_1 = N_B i_B' + N_c i_c - N_L i_L' + N_F i_F$$

$$F_2 = N_B i_B' + N_c i_c + N_L i_L' + N_F i_F$$

$$F_3 = N_B i_B'' - N_c i_c + N_L i_L'' - N_F i_F$$

$$F_4 = N_B i_B'' - N_c i_c - N_L i_L'' - N_F i_F$$

The coercive forces F_2 and F_3 cause saturation during

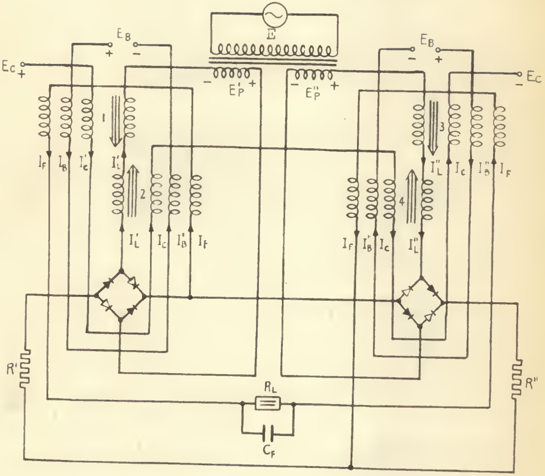


Fig. 9. Equivalent circuit of the amplifier during the nth half-cycle operation with positive signal.

the nth half-cycle.

During the (n + 1)th half-cycle with positive control signal, core 1 will saturate at α_1, core 4 will saturate at α_2, while cores 2 and 3 will remain unsaturated.

During the nth half-cycle with negative control signal, core 3 saturates at α_1, and core 2 saturates at α_2. Cores 1 and 4 remain unsaturated.

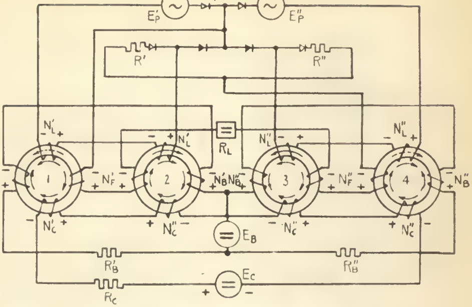
During the (n + 1)th half-cycle with negative control signal, core 4 saturates at α_1, and core 1 saturates at α_2. Cores 2 and 3 remain unsaturated.

In the nth half-cycle positive signal operation, $\phi_2 > \phi_3$. Symmetry considerations shows that a reversal of the polarity of the control signal interchanges the functions of the pairs, thus it changes the polarity of the output. Hence a duodirectional output is obtained.

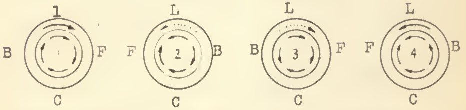
The operation sequence of the magnetic amplifier is illustrated in the following table.

Signal	Power Source	α_1	α_2
Pos. signal	nth half-cycle	2	3
	(n+1)th half-cycle	1	4
Neg. signal	nth half-cycle	3	2
	(n+1)th half-cycle	4	1

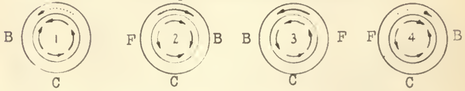
Table 1. Order of Firing of the Four Cores



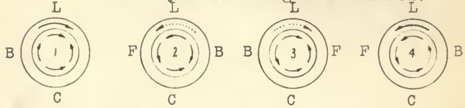
(a). Operation in the nth half-cycle with positive control.



(b). Operation in the (n+1)th half-cycle with positive control.



(c). Nth half-cycle with negative control.



(d). (n+1)th half-cycle with negative control.

Fig. 10. Coercive force direction and flux rising direction. Solid line shows saturated core.

This table implies all operation conditions. Actually, all these four conditions are similar. Thus the analysis does not need go through all conditions. Analysis of any one half-cycle is sufficient. Assume the eddy-current effect is negligible, and that the coercive force is determined from the ampere-turns of the core windings. If high impedance is used in the winding, higher harmonic fluxes will be present in the magnetic core and it will tend to widen the hysteresis loop. Assume a low impedance control winding is used and the even-harmonics effect is negligible.

QUIESCENT CONDITION. - One of the main advantages of a push-pull type magnetic amplifier is very good zero balance, nearly independent to the variations of supply voltage and frequency. If the two single-ended units in the push-pull circuit are not completely matched, the combined output may become unsymmetrical. In order to obtain zero output current, the ratio between the actual resistancees of these two units can be adjusted by means of the slide-wire resistor R_o which makes it easy to get the desired change of I_B'/I_B'' , Thus it is possible to cancel any unsymmetrical elements in these two units. The size of the resistor R_B is chosen to make the cores fire at 90° in the quiescent condition.

In the quiescent condition, $E_c = 0$, and the currents I' and I" are identical; thus no current flows through load resistor R_L and the feedback winding. During the nth half-cycle, the coercive forces are

$$F_1 = F_4 = N_B i_B' - N_L i'$$

$$F_2 = F_3 = N_B i_B'' + N_L i''$$

Cores 2 and 3 saturate at α_0, cores 1 and 4 remain unsaturated. The loop equation of the load circuit is

$$e_p' = \left[-\frac{N_F d\phi_1}{N_L dt} + \frac{N_F d\phi_2}{N_L dt} - \frac{N_F d\phi_3}{N_L dt} + \frac{N_F d\phi_4}{N_L dt} + \frac{d\phi_1}{dt} + \frac{d\phi_2}{dt} \right] N_L$$

Since $F_1 = F_4$, and $F_2 = F_3$, then

$$\frac{d\phi_1}{dt} = \frac{d\phi_4}{dt}, \quad \text{and} \quad \frac{d\phi_2}{dt} = \frac{d\phi_3}{dt}.$$

Equation 19 becomes

$$e_1 + e_2 = e_p$$

where $e_1 = N_L(d\phi_1/dt)$, and $e_2 = N_L(d\phi_2/dt)$

The loop equation of the bias circuit is

$$e_B = (N_B/N_L)e_2 - (N_B/N_L)e_1 = (N_B/N_L)e_3 - (N_B/N_L)e_4 \qquad (20)$$

By Eqs. 19 and 20 the rate of flux change is

$$e_2 = e_3 = \frac{e_p}{2} + \frac{N_L}{2N_B} e_B$$

and

$$e_4 = e_1 = \frac{e_p}{2} - \frac{N_L}{2N_B} e_B.$$

Cores 2 and 3 absorb higher voltage than that of cores 1 and 4 and go to saturation in the nth half-cycle. After these two cores saturate , e_2 and e_3 drop to zero. Then the circuit equations become

$$e_p = i'(2r_f + R) - e_1' - (N_F/N_L)(e_1' - e_4'),$$

$$N_B i_B - N_L i' = 0 ,$$

$$e_B = i_B R_B - (N_B/N_L)e_1'.$$

By symmetry considerations, $e_1' = e_2'$. Solving for e_1', i_L and i_B,

$$i_L = \frac{N_B^2 e_p - N_B N_L e_B}{N_B^2(2r_f + R) - N_L^2 R_B} \qquad (22)$$

$$e_1' = e_4' = \frac{N_L^2 R_B e - N_L N_B (2r_f - R) e_B}{N_B^2(2r_f + R) - N_L^2 R_B} \qquad (23)$$

If rectified bias of the same frequency as the load winding voltage is used, cores 2 and 3 will return to the unsaturated condition at the end of the nth half-cycle.

Althrough there is no signal applied into the amplifier, there will be even-harmonic currents flowing in the control winding in the quiescent condition.

In the quiescent condition, the cores fire at angle α_o, which can be determined by performing the following integration:

$$\int_0^{\frac{\alpha_o}{\omega}} e_1\,dt + \int_{\frac{\alpha_o}{\omega}}^{\frac{\pi}{\omega}} e_1'\,dt - \int_{\frac{\pi}{\omega}}^{\frac{\pi+\alpha_o}{\omega}} e_2\,dt = 0$$

The values of e_1, e_1', and e_2 are given in Eqs. 21, 23.

Figures 15b, c, d illustrate the theoretical waveform of the quiescent current. The quiescent angle is chosen to be 90°.

The average load winding voltage of a saturable core during one-half cycle is

$$V_L = \frac{\omega}{\pi}\int_0^{\frac{\alpha_o}{\omega}} \left(\frac{e_p}{2} + \frac{1}{2}\frac{N_L}{N_B} e_B\right) dt \ . \qquad (24)$$

This is the voltage that the core winding can

withstand for a flux swing from negative to positive saturation during one half-cycle.

$$\text{If } E_p > \frac{\omega}{\pi}\int_0^{\frac{\alpha_0}{\omega}}\left(\frac{e_p}{2} + \frac{1}{2}\frac{N_L}{N_B}e_B\right)dt \ , \qquad (25)$$

the saturable reactors will go to saturation, even without any other magnetizing force.

The maximum allowable uprating factor of the supply voltage can be determined from Eq. 25. The value of e_p under the integral is the normal value of the supplied voltage.

The firing angle may be found by using either the foregoing average flux equation or the following equation:

$$\phi_s - I_B N_B \frac{u'A}{\ell} = \frac{1}{N_L}\int_0^{\frac{\alpha_0}{\omega}}\left(\frac{e_p}{2} + \frac{1}{2}\frac{N_L}{N_B}e_B\right)dt \ . \quad \left(I_B N_B \frac{u'A}{\ell} = \phi_0\right)$$

where ϕ_0 is the preset flux in core 2 at the beginning of the nth half-cycle, ϕ_s is the saturated flux, A and ℓ are the cross section area and the mean length of the saturable core, respectively.

<u>MODE I OPERATION</u>. - As soon as a control signal is applied to the control winding, the identity of the magnetizing forces between these two single-ended units will no longer exist.

During the nth half-cycle operation with positive control, the polarities of core windings are as shown in Fig. 10.

In the period of mode 1 operation, neither core

saturates, the winding impedance is very high and the winding current will be limited to magnetizing current. At the end of the previous half-cycle, core 1 and core 4 are in the saturated condition. Since the bias voltage is a full-wave rectified sine wave of the same frequency and phase as the load winding voltage, the analysis is considerably simplified. The load current and bias current drop to zero simultaneously at the end of one-half cycle. As soon as the resultant mmf of the saturated core drops to zero, the core returns to the unsaturated condition.

In mode 1, the loop equations of the circuit are

$$e_B = (N_B/N_L)(e_2 - e_1) + i_B' R_B \ ,$$

$$e_B = (N_B/N_L)(e_3 - e_4) + i_B'' R_B \ ,$$

$$E_c = i_c R_c - (N_c/N_L)(e_1 - e_2 + e_3 - e_4) \ .$$

During the exciting period, the windings of a magnetic core can tolerate only a small net mmf. The coercive force may be considered equal to zero. Consequently, $i' = 0$, and $i'' = 0$. No current flows into the feedback winding. The following relations then hold;

$$e_B = (N_B/N_L)(e_2 - e_1) - i_B' R_B = (N_B/N_L)(e_3 - e_4) + i_B'' R_B ,$$

$$E_c = (N_c/N_L)(-e_1 + e_2 - e_3 + e_4) + i_c R_c \ .$$

Since a low impedance control circuit is chosen, the a-c component voltage drop of $i_c R_c$ is very small. If this component may be neglected, then $E_c \triangleq i_c R_c$. Hence, an important relation is obtained:

$$e_1 - e_2 + e_3 - e_4 = 0, \text{ or}$$

$$e_2 - e_1 = e_3 - e_4 \quad . \qquad (26)$$

Load winding voltages may be obtained from the circuit loop equations:

$$\begin{aligned}
e_1 &= \frac{e_p}{2} - \frac{1}{2}\frac{N_L e_B}{N_B} - \frac{1}{2}\frac{N_L N_C R_B E_C}{N_B^2 R_C} \ , \\
e_2 &= \frac{e_p}{2} + \frac{1}{2}\frac{N_L e_B}{N_B} + \frac{1}{2}\frac{N_L N_C R_B E_C}{N_B^2 R_C} \ , \\
e_3 &= \frac{e_p}{2} + \frac{1}{2}\frac{N_L e_B}{N_B} - \frac{1}{2}\frac{N_L N_C R_B E_C}{N_B^2 R_C} \ , \\
e_4 &= \frac{e}{2} - \frac{1}{2}\frac{N_L e_B}{N_B} + \frac{1}{2}\frac{N_L N_C R_B E_C}{N_B^2 R_C} \ .
\end{aligned} \qquad (27)$$

From equation (27) we see that the applied control signal produces an unbalance of voltage distribution between these two single-ended units. Since the voltages across the winding represent the rate of flux change within the magnetic core, equation (27) predicts the saturation sequence and modes of operation. The effect of the positive signal in nth half-cycle operation decreases the firing angle of core 2 and increase the firing angle of core 3. The working range of the given circuit is limited by the maximum allowable strength of the control signal. An abnormally strong signal may cause core 2 and core 4 to fire at the same angle and give no output at all.

Using a method similar to that used in quiescent condition, the firing angle may be found by the following relation.

$$\phi_S - (I_B' \ N_B + I_C N_C)\frac{u'A}{\ell} = \frac{1}{\pi N_L}\int_0^{\alpha_1} e_2 \, d(wt) \quad ,$$

or

$$\phi_s - (I_B^n N_B + I_c N_c)\frac{\mu A}{\ell} = \frac{1}{\pi N_L}\left[\int_0^{\alpha_1} e_3\, d(wt) + \int_{\alpha_1}^{\alpha_2} e_s'\, d(wt)\right]$$

The second terms on the left-hand side of these two equations are the preset fluxes in core 2 and core 3 at the beginning of the nth half-cycle. Since ideal square B-H loop material is nonexistent, the flux level may be represented by the coercive force during the exciting period. All the foregoing equations except the last one are derived assuming an ideal square B-H loop.

MODE 2 OPERATION. - The effect of saturation is equivalent to short circuiting a very high impedance. Though the changes of current in the different windings are very complicated there are still some rules which may be applied to the transient phenomena. Since energy cannot be stored in zero time, at the instant of saturation the total energy stored in the magnetic field remains unchanged. In a considerable number of analyses the magnetic amplifier is viewed as an impedance whose magnitude is gradually changed by direct current. During the steady-state the impedance is considered essentially constant. The power transfer phenomenon of the magnetic amplifier is somewhat like the dead-zone switching circuit familiar in nonlinear circuits. We may consider the magnetic amplifier as an impedance which, during steady-state operation, is very high throughout mode 1 operation and very low throughout mode 2 operation. The change from mode 1 to mode 2 is rapid. After core 2 saturates, the

flux in the core of unit 2 will be continuously built up and the impedance of the load winding of unit 2 remains very high. During mode 2 operation, the system equations are

$$N_L i' - N_F i_F - N_c i_c - N_B i_B' = 0,$$

$$N_F i_F + N_c i_c - N_B i_B'' = 0.$$

$$e_p - i'(2r_f + R') - i_F R_L - e_3' - e_4' = 0,$$

$$e_p - i'(r_f + R') + i_F(r_f + R' + R\ ) - 2e_3' - e_1' = 0 \ ,$$

$$(i' - i_F)(r_f + R') - i_F R_L + e_1' + e_3' - e_4' = 0 \ ,$$

$$e_B - i_B'' R_B + (N_B/N_L)(e_3' - e_4') = 0 \ ,$$

$$e_B - i_B' \ R_B + (N_B/N_L) e_1' = 0 \ ,$$

$$E_c + \frac{N_c}{N_L}(e_1' + e_3' - e_4') - i_c' R_c = 0.$$

During mode-2 operation, there is a possibility that some current will bypass the rectifier bridge and the current i' will not be identical to the current i_L.

If $N_L = N_F$, the ampere-turn relations give for i',

$$i' = \frac{N_B}{N_L R_B}\left[2e_B - \frac{N_B}{N_L}(e_3' - e_4' - e_1')\right] .$$

From the loop equations,

$$e_4' = e_p - i'(r_f + \tfrac{1}{2}R') - \tfrac{1}{2} i_F R_L$$

$$e_3' = e_p + \frac{N_L}{N_B} e_B - \tfrac{1}{2} i'\left(3r_f + 2R' + \frac{N_L^2}{N_B^2} R_B\right) + \tfrac{1}{2} i_F (r_f + R') \qquad (28)$$

$$e_1' = \tfrac{1}{2} i_F (r_f + R' + R_L) - \frac{N_L}{N_B} e_B - \tfrac{1}{2} i'\left(\frac{N_L^2}{N_B^2} R_B - r_f - R'\right)$$

Eliminating the unknown voltages, the following equation

is obtained.

$$i_F = \frac{i'\left(5r_f + 3R' - \frac{N_L^2}{N_B^2}R\right) - 2\left(e_p - \frac{N_L}{N_B}e_B\right)}{r_f + R' - R_L} \qquad (29)$$

If the load is not a pure resistance, there will be no linear relation between the load winding current and the feedback current.

When

$$e_1' + e_3' - e_4' = 0\ ,$$

then

$$i_F = \frac{R''}{R'' + R_L}\, i'$$

In this equation, the forward resistance of the rectifier is lumped with the resistance R''.

Although it is possible to solve for load current as a function of the signal current, the result would be very complicated in form.

During mode 2 operation, useful current may flow through the by-pass circuit and the efficiency of the circuit will be reduced. In order to limit the bypass current, the resistance of the ballast resistor should be very high.

Core 1 and core 4 remain unsaturated throughout the nth half-cycle. During this period they are resetting core fluxes.

Since output voltage appears across the load only during mode 2, the half-cyclic average of the load voltage is

$$V_o = \frac{R_L}{\pi}\int_{\alpha_1}^{\alpha_2} I_m \sin wt\, d(wt) = \frac{I_m R_L}{\pi}(\cos\alpha_1 - \cos\alpha_2). \quad (30)$$

During mode 2 operation, the voltages across the rectifier stacks are (Fig. 11a)

$$V_{ab} = V_{cd} = e_p' - e_1' - i_1 r_f$$

$$V_{ad} = V_{bc} = i_1 r_f$$

$$V_{b'c'} = V_{a'd'} = V_{b'a'} = V_{c'd'} = \frac{(i_1 - i_f)}{2} r_f.$$

When

$$e_1' + e_3' - e_4' = i_f R_L$$

no current flows through the by-pass, and $i' = i_f$.

<u>MODE III OPERATION</u>. - After core 2 saturates, core 3 and core 4 still absorb voltage to build up fluxes, and core 3 goes to saturation at angle α_2. After core 3 saturates, the circuit again becomes symmetrical.

The system equations become

$$e_p' = i'(R' + 2r_f) + e_4''$$

$$e_p'' = i''(R'' + 2r_f) + e_1''$$

$$e_B = i_B' R_B - \frac{N_B}{N_L} e_1''$$

$$e_B = i_B'' R_B - \frac{N_B}{N_L} e_4''$$

$$E_c = i_c R_c + \frac{N_c}{N_L}(e_4'' - e_1'')$$

$$N_L i' + N_c i_c - N_B i_B' = 0$$

$$N_L i'' + N_c i_c - N_B i_B'' = 0 \quad .$$

The load winding currents are found as follows:

$$i' = \frac{N_B^2 e_p + N_L N_B e_B}{N_L^2 R_B + N_B^2 (R' + 2r_f)} + \frac{N_L N_c R_B E_c}{N_L^2 R_B R_c + (2N_c^2 R_B - N_B^2 R_c)(R' + 2r_f)} \tag{31}$$

$$i'' = \frac{N_B^2 e_p + N_L N_B e_B}{N_L^2 R_B + N_B^2 (R' + 2r_f)} - \frac{N_L N_c R_B E_c}{N_L^2 R_B R_c + (2N_c^2 R_B - N_B^2 R_c)(R' + 2R_f)} \tag{32}$$

$$i' + i'' = \frac{2N_B^2 e_p + 2N_L N_B e_B}{N_L^2 R_B + (R' + 2r_f) N_B^2} \quad . \tag{33}$$

Equations (32) and (33) illustrate that if the applied signal is very strong, a small current will flow through the load resistor. Since $(E_p + E_B) \gg E_c$, the load current during mode 3 is negligibly small.

Substituting the practical winding data into the load current equation of mode 2;

$$i = \frac{2e_p + 2e_B}{R_B + R_L + 2R'} \quad . \tag{34}$$

Comparing Eq.(34) with Eq.(32), we find that the load winding current will decrease when the system goes into mode 3.

During mode 3 operation, the voltages across the rectifier stacks are (Fig. 11b)

$$V_{ab} = V_{dc} = e' - i' r_f - e_1''$$

$$V_{bc} = V_{ad} = i' r_f$$

$$V_{a'b'} = V_{d'c'} = - i'' r_f$$

$$V_{d'a'} = V_{c'b'} = e_p'' - i'' r_f - e_4'' \quad .$$

As soon as the resultant mmf of the saturated core drops to zero, the system returns to mode 1.

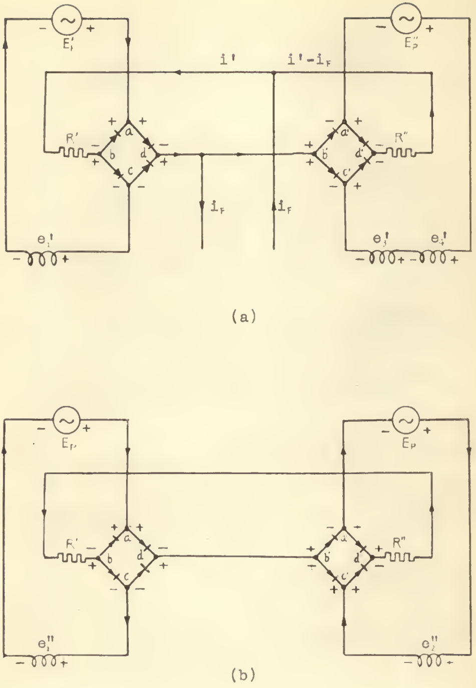


Fig. 11. Voltages across the rectifiers during mode 1 (a) and mode 2 (b) operation.

BLOCK DIAGRAM OF THE PUSH-PULL MAGNETIC AMPLIFIER CIRCUIT. Since magnetic amplifiers are often used in feedback loop systems, they are also represented in block diagram form. A theoretical analysis of the transfer function of a push-pull magnetic amplifier should be based upon very complete information about the saturable cores. The labor involved in a theoretical study of the transfer function would be large. The problem will be approached from fundamental physical concepts which avoid mathematical complexity.

The time constant of the magnetic amplifier is caused by the inherent time delay introduced by the method of control. This delay is always present because the flux level in the magnetic core can only be controlled when the core is unsaturated. An effective way to reduce the time constant is to use a high frequency power supply.

The transient response of the magnetic amplifier depends upon the B-H characteristics. In the exciting region, the operation of the circuit may differ markedly from the theoretical analysis. Using an ideal B-H loop as a basic assumption to derive the transient response and the time constant might cause too large an error. The actual time delay may be several times as large as the analytic result.

By analogy to the block diagram of the single-ended magnetic amplifier, the block diagram of the push-pull

circuit is obtained as shown in Figure 12. The factor K (volts per ampere-turn) is the gain constant, which relates the change in average output voltage to the change in average control ampere-turns. The value of the factor K depends on load resistance, ballast resistance and magnetic core characteristics. For an a-c control signal the factor N_c/R_c should be changed to $N_c'/(1+ST)$; that is, another time constant will be added into the control winding. The actual values of the gain-constant and the time-constant may be obtained by practical measurements.

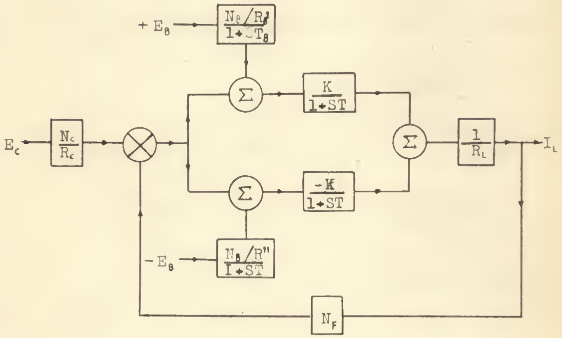


Fig. 12. Block diagram of push-pull magnetic amplifier.

<u>TRANSFER CHARACTERISTICS</u>. - The transfer characteristic of any one single-ended unit used in the given push-pull circuit is shown in Figure 13a. Figure 13b is constructed from two transfer curves of single-ended units. In Figure 13b, from -A to -A is the working range of the push-pull type magnetic amplifier.

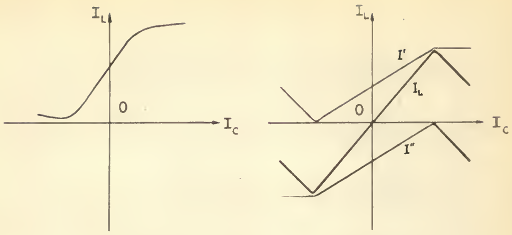
A push-pull magnetic amplifier performs precise measurements and has linear transfer characteristics in the normal working range. The linear transfer characteristics may be obtained even if the transfer characteristic of the component single-ended magnetic amplifier are nonlinear up to second order. The comparison of the transfer characteristics of single-ended units and push-pull magnetic amplifiers is illustrated in Figure 14. In Figure 14, two pairs of parabolic transfer curves of single-ended magnetic amplifiers combine to give linear push-pull output. Mathematically this is easily deduced by applying Taylor's theorem to individual transfer characteristics, and ascertaining that the difference is linear apart from third and higher-order correction terms. For example, the equation of the transfer curve in Figure 14 may be written as

Unit 1, $I' = f(i_B + i_c) = f(i_B) + i_c f'(i_B) + \frac{1}{2} i_c f''(i_B) + \cdots$

Unit 2, $I'' = f(i_B - i_c) = f(i_B) - i_c f'(i_B) + \frac{1}{2} i_c f''(i_B) - \cdots$

The combined transfer characteristics of the push-pull circuit is

$$I' - I'' = 2f'(i_B) - \frac{1}{6} i_c^3 f'''(i_B) .$$



(a) (b)

Fig. 13a. Transfer characteristics of single-ended magnetic amplifier.

13b. Push-pull circuit transfer characteristics.

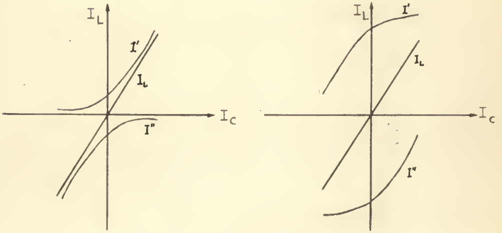


Fig. 14. Linear transfer characteristics of push-pull type magnetic amplifier.

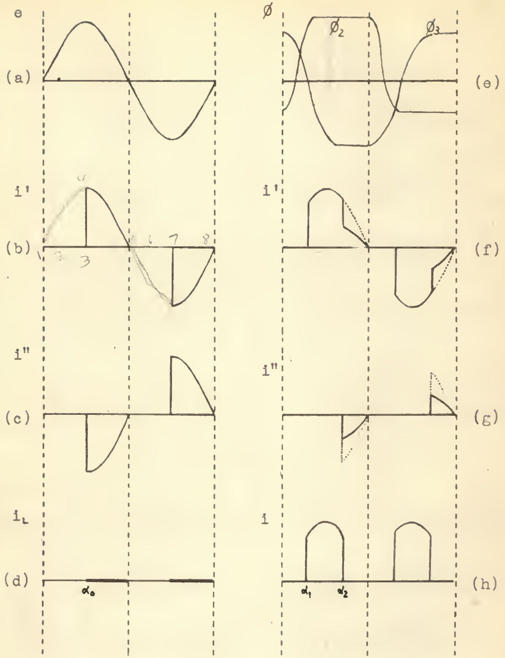


Fig. 15. Mode of operation of the push-pull circuit.

CHAPTER IV

CONCLUSIONS

It has been shown that with the given configuration, one can obtain duodirectional output. Due to the fundamental symmetry between saturable reactors on adjacent half-cycles, it has been possible to limit the analysis to one-half cycle. The characteristics of a single-ended magnetic amplifier and the characteristics of two single-ended units connected in push-pull have been studied. By thinking of the circuit proposed in this paper as a basic rather than a derived circuit, one should be able to understand its limitations and its control and bias requirements without resorting to the connections of the single-ended circuit from which it is derived. The given circuit, in common with most push-pull magnetic amplifier circuits, is very inefficient, but has considerable freedom from effects of supply voltage and frequency variations. If an abnormally strong signal is impressed upon the control circuit of the given configuration the output will decrease and asymptotically approach zero. When this characteristic is objectionable, nonlinear circuit elements may be used to prevent a very strong signal from being applied to the control winding. Ballast resistors are usually selected to be one to two times the value of load resistor. The saturable cores are normally operated

overexcited.

The given circuit is designed to give cancellation of the induced fundamental frequency voltage in the control winding; however, there is a sizable even-harmonic voltage which does not cancel out. In general, the amount of the even-harmonic varies with the amount of signal applied and the load current. Second harmonic voltage may either boost or buck the action of the d-c signal, so when designing a multistage amplifier one should attempt to prevent second harmonics as much as possible.

The analysis was carried out only for a resistive load. When the circuit is applied to an inductive load, the transfer characteristic will no long be a straight line. For a correct analysis, complete information on the F_K-test curve and hysteresis characteristics of the saturable core are necessary, but this would be very complicated. The predication of control requirements and transfer characteristics becomes unavoidably cumbersome and hardly practical. Because the assumed ideal square B-H loop is nonexistent, the arbitrary idealization yields questionable results, but the simplified analysis may help one understand the influence of the circuit elements on operation and to explain some of the intriguing characteristics of the push-pull type magnetic amplifier.

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The author, Mao C. Wang, was born in Hopei, China, on November 21, 1925. His undergraduate studies were pursued at the National Peiyang University, Tientsin, China, from which he received the degree of Bachelor of Science in Electrical Engineering in June, 1948. Following graduation he worked with the Taiwan Alkali Company. In June, 1957, he entered the University of Florida to work for the degree Master of Science in Engineering.

This thesis was prepared under the direction of the chairman of the candidate's supervisory committee and has been approved by all members of the committee. It was submitted to the Dean of the College of Engineering and to the Graduate Council, and was approved as partial fulfillment of the requirements for the degree of Master of Science in Engineering.

August 9, 1957

Joseph Weil
Dean, College of Engineering

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